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SOLUTIONS OF THE TYPE OF TRAVELING WAVES INCLUDING HYPERBOLIC HEAT TRANSFER

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Solutions of the type of traveling waves defining heat transfer and gas motion with account for the relaxation of the heat flow have been obtained and investigated. A comparative analysis of solutions of both types has been carried out. It is shown that, in the case where the relaxation of the heat flow is taken into account, solutions of the type of traveling waves can exist for any relation between the velocity of the traveling wave and the velocity of sound.

Introduction. In many cases where heat-transfer processes are investigated, the heat flow W is defined by the Fourier law:

$$W = W_F = -K \text{ grad } dT .$$

However, the Fourier law can be used only in the case where the mean free path and the mean free time of particles are small as compared to the characteristic space and time scales of change in the temperature of a medium. For example, in a laser plasma, because of the small sizes of the targets exposed to radiation and the high temperatures, the range of electrons and the linear scales of the problem are frequently of the same order of magnitude. In this case, the Fourier model gives overstated heat flows that can be larger than the limited "vacuum" flow W_v arising as a result of the coordinated motion of electrons in one direction. At the moment there are several models allowing one to solve the problem being considered. In the present investigation we will use the model of hyperbolic heat transfer including the heat-flow relaxation $W = -K \text{ grad } T - \tau dW/dt$. This model has a rigorous physical substantiation — it is obtained from the kinetic equations by the Grad method of 13 moments (the deduction of the corresponding equation is given, e.g., in [5]).

The processes of gas motion and heat transfer will be analyzed using solutions of the type of traveling waves. It was shown earlier for models based on the Fourier law that these solutions can be obtained only in the region where the velocity of a traveling wave is larger than the velocity of sound [1]. However, it was established that solutions of the type of traveling waves including the relaxation of a heat flow allow one to consider thermal and gas-dynamic quantities with physical properties changing in a wide range. In this case, both supersonic and subsonic flows can be considered. Unlike the Fourier model, in the model of hyperbolic heat transfer, the desired functions, including the temperature and heat-flow functions, experience discontinuities at the initial stage and in the process of their change with space and time.

Traveling Waves Including a Hyperbolic Heat Transfer in an Immovable Medium. *Formulation of the problem.* The heat transfer with a heat-flow relaxation in an immovable medium is defined by the system of equations in mass variables $m \geq 0$ and $t \geq 0$, constructed in the plane-symmetry approximation,

$$\frac{R}{\gamma - 1} \frac{\partial T}{\partial t} = - \frac{\partial W}{\partial m} , \quad (1)$$

$$W = - K \frac{\partial T}{\partial m} - \tau \frac{\partial W}{\partial t} , \quad K = K_0 T^{a_0} , \quad \tau = \tau_0 T^{a_1} , \quad a_0 > 0 , \quad a_1 > 0 . \quad (2)$$

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Let, in the plane $m = 0$,

$$W(0, t) = W_0 t^g . \tag{3}$$

At $t = 0$,

$$T(m, 0) = 0 , \quad W(m, 0) = 0 . \tag{4}$$

The solution of problem (1)–(4) is self-similar on condition that $a_1 = (a_0 + 2)/(2g + 1)$. The independent variables and desired functions are represented in the dimensionless form

$$s = \frac{m}{(R^{-(a_0+1)} K_0 W_0^{a_0})^{1/(a_0+2)} t^n} , \quad f(s) = \frac{T(m, t)}{(R^{-1} K_0^{-1} W_0^2)^{1/(a_0+2)} t^{n_0}} , \quad \omega(s) = \frac{W(m, t)}{W_0 t^g} , \tag{5}$$

where

$$n = \frac{(g + 1)\, a_0 + 1}{a_0 + 2} > 0 ; \quad n_0 = \frac{2g + 1}{a_0 + 2} = \frac{1}{a_1} .$$

Replacement of variables (5) makes it possible to reduce system (1), (2) to the system of ordinary differential equations for $s \geq 0$

$$\frac{1}{\gamma - 1} \left(n_0 f - n s f' \right) = - \omega' , \quad \omega = - f^{a_0} f' - \hat{\tau}_0 f^{a_1} (g\omega - n s \omega') , \tag{6}$$

where $\hat{\tau}_0 = \tau_0 \left[\frac{W_0^2}{R K_0} \right]^{\frac{a_1}{a_0+2}}$ is a dimensionless constant, and the prime denotes derivative with respect to s. The boundary conditions (3) and (4) in variables (5) have the form

$$\omega(0) = 1 , \tag{7}$$

$$f(s_0) = 0 , \quad \omega(s_0) = 0 , \quad 0 < s_0 \leq \infty . \tag{8}$$

At $\tau = 0$ (Fourier law), problem (1)–(4) has a continuous solution [1]. In the case where $s = s_0$ and $0 < s_0 \leq \infty$, the desired functions $f = f(s)$ and $\omega = \omega(s)$ satisfy conditions (8). At $\tau_0 \neq 0$ [2–4], the functions $T = T(m, t)$ and $W = W(m, t)$ can experience a large discontinuity because of the hyperbolic heat transfer:

$$D = \frac{dm_0}{dt} = n s_0 \left(R^{-(a_0+1)} K_0 W_0^{a_0} \right)^{1/(a_0+2)} t^{n-1} . \tag{9}$$

In the case where a "zero background" (4) is realized downstream of the discontinuity front,

$$\frac{R}{\gamma - 1} T_2 D = W_2 , \quad W_2 D = V_2 , \tag{10}$$

where the parameter V_2 is determined from the following considerations [3, 4]. Let a heat flow is defined as

$$\frac{\partial W}{\partial t} = -\frac{\partial V}{\partial m} - \frac{W}{\tau}, \quad (11)$$

where the function $V = V(m, t)$ satisfy the equation

$$\frac{\partial V}{\partial m} = \frac{K(T)}{\tau(T)} \frac{\partial T}{\partial m} = \frac{dV}{dT} \frac{\partial T}{\partial m}. \quad (12)$$

Using (2), we can write the term $\frac{dV}{dT}$ in (12) in the form

$$\frac{dV}{dT} = \frac{K_0}{\tau_0} T^{a_0 - a_1}. \quad (13)$$

Upon integrating (13) at $a_0 - a_1 + 1 \neq 0$, we obtain $V = \frac{K_0}{\tau_0 (a_0 - a_1 + 1)} T^{a_0 - a_1 + 1} + C_0$. Let $V = 0$ at $T = 0$ and $a_0 - a_1 + 1 > 0$; then $C_0 = 0$ and, consequently, $V_2 = \frac{K_0}{\tau_0 (a_0 - a_1 + 1)} T_2^{a_0 - a_1 + 1}$. Formulas (10) in variables (5) will take the form

$$\frac{1}{\gamma - 1} f_2 n s_0 = \omega_2, \quad \omega_2 n s_0 = \frac{f_2^{a_0 - a_1 + 1}}{\hat{\tau}_0 (a_0 - a_1 + 1)}. \quad (14)$$

Construction of solution of the type of traveling waves. Let us assume that the desired functions obtained as a result of the self-similar solution of problem (1)–(4) can be represented in the form of a traveling wave:

$$T(m, t) = \tilde{T}(\bar{D}t - m), \quad W(m, t) = \tilde{W}(\bar{D}t - m), \quad \bar{D} = \text{const}. \quad (15)$$

Because of the self-similarity of the indicated solution, it may be assumed that $F(\bar{D}t - m) = \bar{F}(m/t)$, i.e., that $n = 1$ in the expression for the variable s. Then we obtain that $g = 1/a_0$, $n_0 = 1/a_0$, and $a_0 = a_1$.

At $n = 1$, $D = \text{const}$. Let $\bar{D} = D$; then the system of equations (6) will take the form

$$\frac{1}{\gamma - 1}\left(\frac{1}{a_0} f - s f'\right) = -\omega', \quad \omega = -f^{a_0} f' - \hat{\tau}_0 f^{a_0}\left(\frac{1}{a_0}\omega - s\omega'\right), \quad 0 \le s \le s_0, \quad 0 < s_0 \le \infty. \quad (16)$$

The solution of system (16) satisfies the boundary condition (7) and directly the equation for the "zero background" (8) or, if, at $s = s_0$, a large discontinuity takes place, conditions (10), which, in variables (5), at $n = 1$ and $a_0 = a_1$, take the form:

$$\omega_2 = \frac{1}{\gamma - 1} f_2 s_0, \quad s_0^2 = \frac{\gamma - 1}{\hat{\tau}_0}. \quad (17)$$

Equation (1) can be written as

$$\frac{R}{\gamma - 1} D \frac{dT}{dy} = \frac{dW}{dy}, \quad y = \bar{D}t - m. \quad (18)$$

The solution of Eq. (18) at condition (4) and the possible condition of discontinuity of the desired quantities (10) has the form $W = RDT/(\gamma - 1)$, which, in variables (5), can be represented as

$$\omega = \frac{1}{\gamma - 1} f s_0 . \tag{19}$$

Using relation (19), we obtain $\omega' = \frac{s_0}{\gamma - 1} f'$. In this case, Eq. (16) can be written in the following form:

$$\frac{1}{\gamma - 1}\left(\frac{1}{a_0} f - s f'\right) = -\frac{1}{\gamma - 1} s_0 f', \tag{20}$$

$$\frac{s_0 f}{\gamma - 1} = -f^{a_0} f' - \frac{1}{\gamma - 1} \hat{\tau}_0 s_0 f^{a_0}\left(\frac{1}{a_0} f - s f'\right). \tag{21}$$

Rearrangement of Eqs. (20) and (21) gives the equation $f^{a_0 - 1}(\tau_0 s_0^2 - \gamma + 1) f' = s_0$, which, upon integration, takes the form

$$f^{a_0} = \frac{s_0 a_0}{\hat{\tau}_0 s_0^2 - \gamma + 1} s + C . \tag{22}$$

It follows from (22) that the condition $s_0^2 = (\gamma - 1)/\hat{\tau}_0$ can be formally fulfilled only at $f = \infty$. Therefore, at both $\hat{\tau}_0 = 0$ and $\hat{\tau}_0 \neq 0$, the solution being considered should be continuous at a finite value of *f*. From the condition $f(s_0) = 0$ we obtain that $C = (-s_0^2 a_0)/(\tau_0 s_0^2 - \gamma + 1)$. Relation (22) takes the form

$$f = \left(\frac{s_0 a_0}{\gamma - 1 - \hat{\tau}_0 s_0^2}\right)^{1/a_0} (s_0 - s)^{1/a_0} . \tag{23}$$

At $\hat{\tau}_0 = 0$, relation (23) and the expression for the heat flow $\omega = f s_0/(\gamma - 1)$ are identical to the corresponding formulas presented in [1] for the case of the Fourier law. It follows from (23) that the inequality $\hat{\tau}_0 < (\gamma - 1)/s_0^2$ should be fulfilled, i.e., the parameter $\hat{\tau}_0$ proportional to the relaxation time of the heat flow $\hat{\tau}_0$ is limited from above.

The boundary condition (7) leads to the relation $s_0/f(0)/s(\gamma - 1) = 1$, or, in view of (23), to the relation

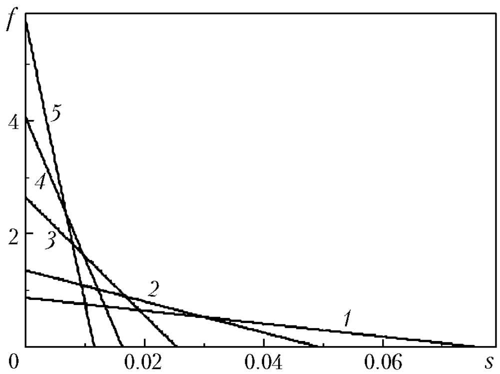
$$\frac{1}{\gamma - 1} s_0 \left(\frac{s_0^2 a_0}{\gamma - 1 - \hat{\tau}_0 s_0^2}\right)^{1/a_0} = 1 . \tag{24}$$

Rearrangement of expression (24) gives the relation

$$s_0^2 \left(a_0 s_0^{a_0} + (\gamma - 1)^{a_0} \hat{\tau}_0\right) = (\gamma - 1)^{a_0 + 1} . \tag{25}$$

Using the algebraic formulas (24) and (25), we can determine the parameter s_0, i.e., the coordinate characterizing the depth of heating at different values of $\hat{\tau}_0$ limited by the condition $0 \leq \hat{\tau}_0 < (\gamma - 1)/s_0^2$.

Figure 1 presents the distribution of the function $f = f(s)$ at $f = \frac{3 s_0}{2 - 3\tau_0 s_0^2}(s_0 - s)$ for the coordinate $s = s_0$ at different values of $\hat{\tau}_0$. An analysis has shown that, with increase in $\hat{\tau}_0$, the absolute temperature increases and the depth of heating decreases.



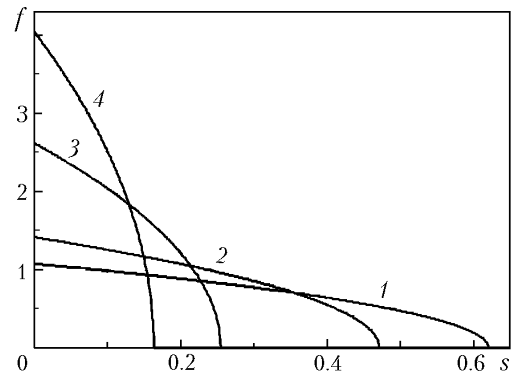


Fig. 1. Dependence of the dimensionless temperature on s ($\gamma = 5/3$, $a_0 = 1$): 1) $\hat{\tau}_0 = 0$ and $s_0 = 0.763$; 2) 2 and 0.493; 3) 10 and 0.253; 4) 25 and 0.1625; 5) 50 and 0.11527.

Fig. 2. Dependence of the dimensionless temperature on s ($\gamma = 5/3$, $a_0 = 2$): 1) $\hat{\tau}_0 = 0$ and $s_0 = 0.624$; 2) 2 and 0.473; 3) 10 and 0.258; 4) 25 and 0.165.

At $a_0 = 2$, from (25) we obtain the biquadratic equation $2s_0^4 + (\gamma - 1)^2 \hat{\tau}_0 s_0^2 - (\gamma - 1)^3 = 0$, the solution of which has the form

$$s_0^2 = \frac{1}{4} (\gamma - 1)^2 \left[\sqrt{\hat{\tau}_0^2 + \frac{8}{\gamma - 1}} - \hat{\tau}_0 \right].$$

Figure 2 shows the distribution $f = f(s)$ at $f = \left(\frac{6s_0}{2 - 3\hat{\tau}_0 s_0^2} \right)^{1/2} (s_0 - s)^{1/2}$ for the coordinate $s = s_0$ at different values of $\hat{\tau}_0$. Let us write relation (23) in the dimensional form:

$$T = \left(\frac{R^{-1} K_0^{-1} a_0 D}{\gamma - 1 - \frac{\tau_0 R}{K_0} D^2} \right)^{1/a_0} (Dt - m)^{1/a_0} . \tag{26}$$

In the case of relaxation of a heat flow ($\tau \neq 0$), the mass velocity of propagation of the thermal disturbances is determined by the relation $D_T = \sqrt{\frac{(\gamma - 1)K}{\tau R}}$ (see, e.g., [2–5]). In the case being considered, where $a_1 = a_0$, we can write that $D_T^2 = K_0(\gamma - 1)/\tau_0 R$ and formula (26) takes the form

$$T = \left(\frac{K_0^{-1} R a_0 D}{(\gamma - 1) \left(1 - \frac{D^2}{D_T^2} \right)} \right)^{1/a_0} (Dt - m)^{1/a_0} . \tag{27}$$

Function (27) is determined in the region of $m \leq DT$, where $D_T^2 > D^2$. At $m \leq Dt$, the heat flow is defined as $W = RDT/(\gamma - 1)$. At $m \geq DT$, $T \equiv$ and $W \equiv 0$.

Hyperbolic Heat Transfer in the Form of Traveling Waves Including a Gas Motion. *Formulation of the problem.* The system of gas-dynamic equations including a hyperbolic heat transfer is written in Lagrange mass variables m and t in the plane-symmetry approximation

$$\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)=\frac{\partial v}{\partial m}\,,\quad \frac{\partial v}{\partial t}=-\frac{\partial p}{\partial m}\,,\quad \frac{\partial}{\partial t}\left(\varepsilon+\frac{1}{2}v^2\right)=-\frac{\partial}{\partial m}(pv+W)\,,\quad p=p(\rho,T)\,,\quad \varepsilon=\varepsilon(\rho,T)\,. \tag{28}$$

As in the previous case, the heat flow W is defined with account for its relaxation:

$$W=-K\frac{\partial T}{\partial m}-\tau\frac{\partial W}{\partial t}\,,\quad K=K(\rho,T)\,,\quad \tau=\tau(\rho,T)\,. \tag{29}$$

In the case where the relaxation of the heat flow is taken into account, the system of equations (28), (29) admits a large discontinuity of the desired quantities, including the temperature function $T = T(m, t)$ and the function of the heat-flow density $W = W(m, t)$.

The relations for the desired quantities at the possible discontinuity front will be written with the use of the auxiliary function $V = V(m, t)$ satisfying the equation

$$\frac{\partial V}{\partial m}=\frac{K(\rho,T)}{\tau(\rho,T)}\frac{\partial T}{\partial m}\,. \tag{30}$$

With the use of (30) at $\tau \neq 0$, Eq. (29) can be represented in the form

$$\frac{\partial W}{\partial t}=-\frac{\partial V}{\partial m}-\frac{W}{\tau}\,. \tag{31}$$

Let $m = m_j = m_j(t)$ is the wake of the discontinuity surface in the plane (m, t), then $D = dm_j/dt$ is the mass velocity of the discontinuity front. The relation for the desired quantities at the discontinuity front will be obtained by formal integration of the gas-dynamic equations much as this was done in [6]. Let us integrate (28) and (31) over the small region (reducing to the zero volume) of change in the independent variables m and t, including a discontinuity line:

$$\left(\frac{1}{\rho_2}-\frac{1}{\rho_1}\right)D=v_1-v_2\,,\quad (v_2-v_1)\,D=p_2-p_1\,,$$

$$\left(\varepsilon_2+\frac{1}{2}v_2^2-\frac{1}{2}v_1^2-\varepsilon_1\right)D=p_2v_2-p_1v_1+W_2-W_1\,,\quad (W_2-W_1)\,D=V_2-V_1\,. \tag{32}$$

Let the velocity of the piston and the temperature regime at its surface in the plane $m = 0$

$$v(0,t)=v_*(t)\,,\quad T(0,t)=T_*(t) \tag{33}$$

be selected such that the gas-dynamic and thermal disturbances propagate along the front

$$v_1=0\,,\quad T_1=0\,,\quad p_1=0\,,\quad W_1=0\,,\quad V_1=0\,,\quad \rho_1=\rho_0=\text{const} \tag{34}$$

in the form of a traveling wave; i.e., for each desired function F we can write the following relations: $F(m, t) = \hat{F}(Dt - m)$, D = const. In the subsequent discussion we will use the following equations of ideal-gas state:

$$p = R\rho T , \quad \varepsilon = \frac{RT}{\gamma - 1} . \tag{35}$$

Let

$$K = K_0 T^{a_0} \rho^{b_0} , \quad \tau = \tau_0 T^{a_1} \rho^{b_1} , \quad a_0 \geq a_1 > 0 . \tag{36}$$

We will represent the independent variables and desired functions in the dimensionless form

$$x = \frac{Dt - m}{K_0 D^{2a_0 - 1} R^{-(a_0+1)} \rho_0^{-2a_0 + b_0}} , \quad \eta = \eta (x) = \frac{\rho_0}{\rho (m, t)} ,$$

$$\alpha = \alpha (x) = \frac{v (m, t)}{D \rho_0^{-1}} , \quad \beta = \beta (x) = \frac{p (m, t)}{D^2 \rho_0^{-1}} , \quad f = f (x) = \frac{RT (m, t)}{D^2 \rho_0^{-2}} , \tag{37}$$

$$\omega = \omega (x) = \frac{W (m, t)}{D^3 \rho_0^{-2}} , \quad \hat{V} = \hat{V} (x) = \frac{V (m, t)}{D^4 \rho_0^{-2}} .$$

Equations (28), (29), and (31) are reduced with the use of (37), (35), and (36) to the ordinary differential equations for x

$$\frac{d\eta}{dx} = - \frac{d\alpha}{dx} , \quad \frac{d\alpha}{dx} = \frac{d\beta}{dx} , \quad \frac{d}{dx} \left(\frac{f}{\gamma - 1} + \frac{\alpha^2}{2} \right) = \frac{d}{dx} (\alpha\beta + \omega) , \tag{38}$$

$$\frac{d\hat{V}}{dx} = \frac{1}{\varphi_0} f^{a_0 - a_1} \eta^{-b_0 + b_1} \frac{df}{dx} , \tag{39}$$

$$\frac{d\omega}{dx} = \frac{d\hat{V}}{dx} - \frac{1}{\varphi_0} \omega f^{-a_1} \eta^{b_1} , \tag{40}$$

where $\beta = \dfrac{f}{\eta}$ and $\varphi_0 = \dfrac{\tau_0 R^{a_0 - a_1 + 1}}{K_0 D^{2(a_0 - a_1 - 1)} \rho_0^{-2(a_0 - a_1) + b_0 - b_1}}$ is a dimensionless constant.

Upon integration of (38) we obtain

$$\eta = C_0 - \alpha , \quad \alpha = C_1 + \beta , \quad \frac{f}{\gamma - 1} + \frac{\alpha^2}{2} - \beta\alpha - \omega = C_2 . \tag{41}$$

Let us determine the position of the front of a traveling wave by the quantity $m = Dt$ in variables (37) — by the coordinate $x = 0$. To a disturbed medium corresponds the region of $m \leq Dt$, i.e., the region of $x \geq 0$. In the case of a large discontinuity at $\tau \neq 0$, the velocity of propagation of the traveling-wave front will be assumed to be equal to the velocity of discontinuity propagation. The relations for the desired quantities at the discontinuity front (32) and (34) in variables (37) will take the form

$$\alpha_1 = f_1 = \beta_1 = \omega_1 = \hat{V}_1 = 0 , \quad \eta_1 = 1 , \tag{42}$$

$$\alpha_2 = 1 - \eta_2\,, \quad \beta_2 = \alpha_2 = 1 - \eta_2\,, \quad \omega_2 = \frac{f_2}{\gamma - 1} + \frac{\alpha_2^2}{2} - \beta_2\alpha_2\,, \quad \hat{V}_2 = \omega_2\,. \tag{43}$$

Rearrangement of the dimensionless functions of temperature f_2 and heat flow ω_2 gives

$$f_2 = \eta_2\,(1 - \eta_2)\,, \quad \omega_2 = \frac{1}{2}\,\frac{\gamma + 1}{\gamma - 1}\,(1 - \eta_2)\left(\eta_2 - \frac{\gamma - 1}{\gamma + 1}\right). \tag{44}$$

In this case, $C_0 = 1$, $C_1 = 0$, and $C_2 = 0$ in accordance with (43) and (44), and formulas (41) take the form

$$\alpha = 1 - \eta\,, \quad \beta = 1 - \eta\,, \quad f = \eta\,(1 - \eta)\,, \quad \omega = \frac{1}{2}\,\frac{\gamma + 1}{\gamma - 1}\,(1 - \eta)\left(\eta - \frac{\gamma - 1}{\gamma + 1}\right). \tag{45}$$

Functions (45) also directly satisfy conditions (42).

With the use of (39) and (45), the equations for the functions $\hat{V} = \hat{V}(x)$ in the region of $x \geq 0$ can be written as

$$\frac{d\hat{V}}{d\eta} = \frac{1}{\varphi_0}\,\eta^{a_0 - a_1 - b_0 + b_1}\,(1 - \eta)^{a_0 - a_1}\,(1 - 2\eta)\,. \tag{46}$$

It would appear reasonable that the following boundary condition can be set for (46): $\eta = 1$ ($f = 0$), $\hat{V} = 0$. On rearrangement with the use of (39), (40), and (45), we obtain an equation for determining the dimensionless specific volume η in the region of $x \geq 0$, $0 \leq \eta \leq 1$:

$$\frac{d\eta}{dx} = \frac{1}{2}\,\frac{\gamma + 1}{\gamma - 1}\,\frac{\eta - \dfrac{\gamma - 1}{\gamma + 1}}{\eta^{a_0 - b_0}\,(1 - \eta)^{a_0 - 1}\,(1 - 2\eta) + \varphi_0\,\dfrac{\gamma + 1}{\gamma - 1}\left(\eta - \dfrac{\gamma}{\gamma + 1}\right)\eta^{a_1 - 1}\,(1 - \eta)^{a_1 - 1}}\,. \tag{47}$$

Characteristic properties of the solution. We now consider a number of properties of the solution of Eq. (47) at $a_0 > 0$ and $a_1 > 0$.

1. It will be assumed that $\eta = 1 - \tilde{\eta}$, where $\tilde{\eta} > 0$, is a small quantity in the neighborhood of $x = 0$, $\eta = 1$.

At $\varphi_0 = 0$ (the heat flow is adequately defined by the Fourier law), the solution of the problem being considered is continuous. At $x = 0$, it is required that $\eta = 1$. Near $x = 0$, we obtain that $\eta = 1 - \left(\dfrac{a_0}{\gamma - 1}\right)^{1/a_0} x^{1/a_0}$.

Let $\varphi_0 \neq 0$.

a) Conserving the principal terms in (47) at $a_0 = a_1$ and $\varphi_0 \neq \gamma - 1$, we obtain $\tilde{\eta}^{\,a_0} = \dfrac{a_0}{\gamma - 1 - \varphi_0}\,x$. The variable $\tilde{\eta}$ can reach the value $\tilde{\eta} = 1$ in the region of $x > 0$ only at a limited value of the parameter φ_0: $\varphi_0 < \gamma - 1$. In this case, in the neighborhood of $x = 0$, $\eta = 1$, the solution of Eq. (47) is also continuous. The asymptotic solution in the neighborhood of $x = 0$, $\eta = 1$ has the form $\eta = 1 - \left(\dfrac{a_0}{\gamma - 1 - \varphi_0}\right)^{1/a_0} x^{1/a_0}$.

b) Concerving the principal terms in Eq. (47) at $a_0 = a_1$ and $\varphi_0 = \gamma - 1$, we obtain $d\tilde{\eta}/dx = \tilde{\eta}^{\,a_0}$ and, consequently, $\tilde{\eta} = 1 - [(a_0 + 1)x]^{1/(a_0+1)}$ in the region of $x > 0$ near $x = 0$. Thus, in the case where $a_0 = a_1$, the solution of the problem being considered is continuous at $\varphi_0 \leq \gamma - 1$ ($\eta(0) = 1$), and this solution should be discontinuous at $\varphi_0 > \gamma - 1$ ($\eta(0) = \eta_2$, $0 < \eta_2 < 1$).

c) Let $a_0 > a_1 > 0$ and $\varphi_0 \neq 0$. Equation (47) can be written with accuracy to the principal term as $\frac{d\tilde{\eta}}{dx} = \frac{1}{\tilde{\eta}^{a_1-1}}$, whence it follows that $\frac{1}{a_1}\tilde{\eta}^{a_1} = -x$. It follows from this equation that the function $\tilde{\eta}$ can formally reach the value of $\tilde{\eta} = 0$ ($\eta = 1$) only in the region of $x < 0$, which is contradictory to the physical sense. This means that, at $x = 0$ ($m = Dt$), the function $\tilde{\eta} = \rho_0/\rho(m, t)$ necessarily experiences a discontinuity: it is required that $\eta(0) = \eta_2 < 1$. At $x = 0$, the other desired functions (45) have a discontinuity too.

2. We now consider the asymptotics of the solution of Eq. (47) at $\eta \to (\gamma - 1)/(\gamma + 1)$, $1 < \gamma < 3$. As is known, in the case where $\varphi_0 = 0$, $\eta \to (\gamma - 1)/(\gamma + 1)$ at $x \to -\infty$ (see [1, 7]). The solution has a physical meaning only at η ranging from 0.5 to 1 in the finite interval of change in the independent variable $0 \le x \le x_0$, $\eta(x_0) = 0.5$ and, consequently, during a finite time: $0 \le t \le t(x_0)$. It was shown in [1, 7] that, in the indicated region of change in x and η and, correspondingly, in m, t, and ρ, heat is transferred with a supersonic velocity: the condition $D > \rho C_\gamma$, where $C_\gamma = \sqrt{RT}$, is fulfilled. At $\eta = 0.5$, we have $D = \rho C_\gamma$.

It is assumed that in the neighborhood of $\eta = \frac{\gamma - 1}{\gamma + 1}$, $\eta = \frac{\gamma - 1}{\gamma + 1} + z$, where $z > 0$ is a small quantity. On rearrangement, Eq. (47) takes, with accuracy to the principal terms, the form

$$\frac{dz}{dx} = \frac{B_0}{A_0 - \varphi_0} z , \tag{48}$$

where $A_0 = \frac{(2(\gamma - 1))^{a_0 - a_1}(\gamma - 1)^{b_1 - b_0 + 1}(3 - \gamma)}{(\gamma + 1)^{2(a_0 - a_1) + b_1 - b_0 + 1}}$ and $B_0 = \frac{(\gamma + 1)^{2a_1 - b_1}}{2^{a_1}(\gamma - 1)^{a_1 - b_1}}$ are positive constants and $\varphi_0 \neq A_0$. Equation (48) is solved as

$$x = \frac{A_0 - \varphi_0}{B_0} \ln C \, |z| , \tag{49}$$

where $z = \eta - \frac{\gamma - 1}{\gamma + 1}$. Let

$$A_0 = \varphi_0^{\text{cr}} . \tag{50}$$

In this case, the distribution $\eta = \eta(x)$ changes its direction along x. It follows from (49) that $\eta \to \frac{\gamma - 1}{\gamma + 1}$ at $x \to -\infty$ in the case where $\varphi_0 < \varphi_0^{\text{cr}}$ and at $x \to +\infty$ in the case where $\varphi_0 > \varphi_0^{\text{cr}}$.

Thus, in the case of hyperbolic heat transfer, unlike the cases of gas motion and heat transfer in the form of traveling waves by the Fourier law ($\varphi_0 = 0$), Eq. (47) and equations for the other desired functions (45) can have solutions throughout the range of change in the independent variable $x \ge 0$ ($m \ge 0$ at any $t \ge 0$).

Let us consider a number of concrete examples of solving Eq. (47) at different values of the parameter $\varphi_0 \ge 0$.

Analytical solutions at $a_0 = a_1$, $b_0 = b_1$. In this case, Eq. (47) takes the form

$$\frac{d\eta}{dx} = \frac{1}{2}\frac{\gamma + 1}{\gamma - 1} \frac{\eta - \frac{\gamma - 1}{\gamma + 1}}{\eta^{a_0 - b_0}(1 - \eta)^{a_0 - 1}\left[1 - 2\eta + \varphi_0 \frac{\gamma + 1}{\gamma - 1}\left(\eta - \frac{\gamma}{\gamma + 1}\right)\right]} . \tag{51}$$

In the subsequent discussion we assume that $\gamma = 5/3$. Let $a_0 = 1$ and $b_0 = 1$. Upon integrating (51) we obtain

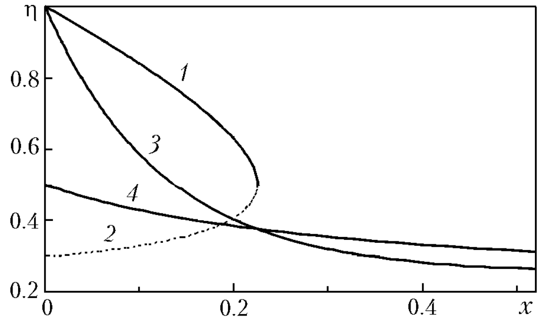


Fig. 3. Profiles of the function $\eta = \eta(x)$, $x \geq 0$ at $a_0 = a_1 = b_0 = b_1 = 1$, $\gamma = 5/3$ and different values of φ_0: $\varphi_0 = 0$ (1), 0 (the portion of curve 1 having no physical meaning) (2), 0.5 (3), and 1 (4).

$$x = (2\varphi_0 - 1)\,\eta + \frac{1}{4}(1 - 3\varphi_0)\ \ln \left|\eta - \frac{1}{4}\right| + C\,. \tag{52}$$

a) At $\varphi_0 = 0$, the constant C is determined from the condition $\eta = 1$ at $x = 0$. In this case, formula (52) takes the form $x = 1 - \eta + \frac{1}{4}\ln\frac{4\eta - 1}{3}$. This solution has a physical meaning only in the finite range of change in the independent variable $0 \leq x \leq x_0 \approx 0.225$. The function $\eta = \eta(x)$ decreases with increase in x as long as $\eta(x_0) = 0.5$. At the point $x = x_0$, $\eta(x_0) = 0.5$ we have $d\eta/dx = \infty$; i.e., this point is a point of rotation of the integral curve $\eta = \eta(x)$. The region where $\eta < 0.5$ has no a physical meaning: $\eta \to 0.25$ at $x \to -\infty$.

b) As was shown above, the solution of the problem being considered is continuous at $\varphi_0 \leq \gamma - 1 = 2/3$. At $a_0 = 1$, $b_0 = 1$, and $\gamma = 5/3$, from (50) it follows that $\varphi_0^{\text{cr}} = 1/3$. Consequently, $\eta \to \frac{\gamma - 1}{\gamma + 1} = \frac{1}{4}$ at $x \to \infty$ in the case where $\varphi_0 > 1/3$. Let $\varphi_0 = 0.5$. The solution of Eq. (52) is continuous. When the constant C in (52) is determined at $\varphi = 0.5$ and $\eta(0) = 1$, $x = -\frac{1}{8}\ln\frac{4\eta - 1}{3}$ or, in the explicit form, $\eta = \frac{1}{4}[1 + \exp(-8x)]$. The problem being considered has a solution in the case where $0 \leq x \leq \infty$ and $1 \geq \eta \geq 0.25$. In this case, at the point $x = 0$, by analogy with the case where $\varphi_0 = 0$, the solution is continuous $(\eta(0) = 1)$.

c) If $\varphi_0 > 2/3$, a discontinuity should exist at $x = 0$. When the values of the parameters a_0 and b_0 fall within the range being considered, Eq. (46) takes the form $\frac{d\tilde{V}}{d\eta} = \frac{1}{\varphi_0}(1 - 2\eta)$. The solution of this equation at the boundary condition $\eta = 1$, $\tilde{V} = 0$ has the form $\tilde{V} = \frac{1}{\varphi_0}\eta\,(1 - \eta)$. Using the relation for the discontinuity front $\tilde{V}_2 = \omega_2$, we obtain that $\eta_2 = \frac{\varphi_0}{2(2\varphi_0 - 1)}$. Let $\varphi_0 = 1$, then $\eta_2 = 0.5$. Formula (52) determining the implicit expression for the function $\eta = \eta(x)$ in the region where $x \geq 0$ and $0.5 \geq \eta \geq 0.25$ can be written as $x = -0.5(\ln(4\eta - 1) + 1 - 2\eta)$.

Figure 3 presents profiles of the function $\eta = \eta(x)$ at $x \geq 0$. At the values of $\varphi_0 \neq 0$ selected, the function $\eta = \eta(x)$ exists in the region where $0 \leq x \leq \infty$; consequently, $t > 0$ at $\eta(0) = 1$ as well as in the case where, at $x = 0$, there takes place a large discontinuity: $\eta(0) = \eta_2 < 1$. At $\varphi_0 = 1$ (Fourier law), the solution has a physical meaning only in the region where $0 \leq x = x_0 \approx 0.225$.

Some examples of solutions at $a_0 > a_1 > 0$. The characteristics of the traveling waves including a heat-flow relaxation experience a large discontinuity at $x = 0$.

Let $a_0 - a_1 = 1$, $b_1 = 1$, and, once again, $\gamma = 5/3$. In this case, Eq. (46) will take the form

$$\frac{d\hat{V}}{d\eta} = \frac{1}{\varphi_0}\eta\,(1 - 3\eta + 2\eta^2)\,. \tag{53}$$

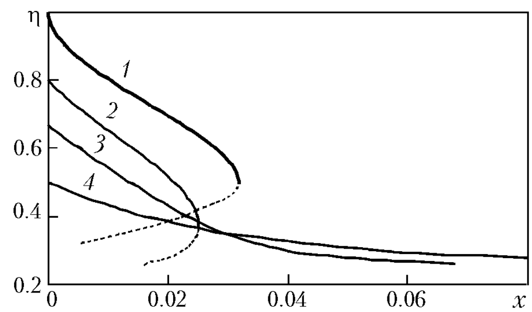


Fig. 4. Distribution of the specific volume along the variable x at a_0 = 2, a_1 = 1, and different values of φ_0: 1) φ_0 = 0 and $\eta(0)$ = 1; 2) 16/275 and 0.8; 3) 4/45 and 2/3; 4) 1/8 and 0.5.

Integration of (53) gives $\tilde{V} = \frac{1}{2\varphi_0}\eta^2\ (1-\eta)^2$. From the condition for the discontinuity front, where x = 0, $0.25 < \eta_2 < 1$, and γ = 5/3, it follows that

$$\varphi_0 = \frac{\eta_2^2\,(1-\eta_2)}{4\eta_2 - 1}\,. \tag{54}$$

It follows from (50) that the "critical" value of the parameter φ_0 is determined by φ_0^{cr} = 0.0625. Let us consider Eq. (47) at a_0 = 2, a_1 = 1, $b_0 = b_1$ = 1, and γ = 5/3

$$\frac{d\eta}{dx} = \frac{2\left(\eta - \frac{1}{4}\right)}{\eta\,(1-\eta)\,(1-2\eta) + 4\varphi_0\left(\eta - \frac{5}{8}\right)}\,. \tag{55}$$

This equation is solved as

$$x = C + \frac{3}{4}\left(\frac{1}{16} - \varphi_0\right)\ln\left|\eta - \frac{1}{4}\right| + \frac{1}{3}\left(\eta - \frac{1}{4}\right)^3 - \frac{3}{8}\left(\eta - \frac{1}{4}\right)^2 - 2\left(\frac{1}{32} - \varphi_0\right)\left(\eta - \frac{1}{4}\right). \tag{56}$$

Figure 4 presents a number of profiles of the function $\eta = \eta(x)$ at different values of the parameter $\eta = \eta(0)$ and the dimensionless constant φ_0. In this figure, curve 1 obtained for a heat flow defined by the Fourier law ($\eta(0) = 1$, $\varphi_0 = 0$) and hyperbolic-heat-transfer profiles having a discontinuity at $x = 0$ are presented. The following boundary conditions were set: $\eta(0) = \eta_2$, $0.25 < \eta_2 < 1$. The values of the parameters were determined by formula (54) for different values of φ_0: η_2 = 0.8, $\varphi_0 = 16/275 \simeq 0.05818 < \varphi_0^{\text{cr}}$ (curve 2); η_2 = 2/3, $\varphi_0 \simeq 4/45 \simeq 0.08889 > \varphi_0^{\text{cr}}$ (curve 3), and η_2 = 0.5, $\varphi_0 = 0.125 > \varphi_0^{\text{cr}}$ (curve 4). In the two last-mentioned cases, the solution of the problem being considered exists in the region where $0 \le x \le \infty$ and $\eta_2 > \eta \ge \frac{\gamma - 1}{\gamma + 1} = \frac{1}{4}$, i.e., throughout the range of change in the variable x. The function $\eta = \eta(x)$, having a discontinuity at $x = 0$, for which, however, $\varphi_0 < \varphi_0^{\text{cr}}$ (see curve 2), has, by analogy with the Fourier law ($\varphi_0 = 0$), a physical meaning only in a finite range of change in x. (The values of $\eta = \eta(x)$ that have no a physical meaning are shown in Figs. 3 and 4 by a dotted line.)

Figure 5 presents profiles of the functions

$$\psi = \psi\,(x) = \frac{D}{\rho\sqrt{RT}} = \frac{\eta}{\sqrt{f}} = \sqrt{\frac{\eta}{1-\eta}}\,, \tag{57}$$

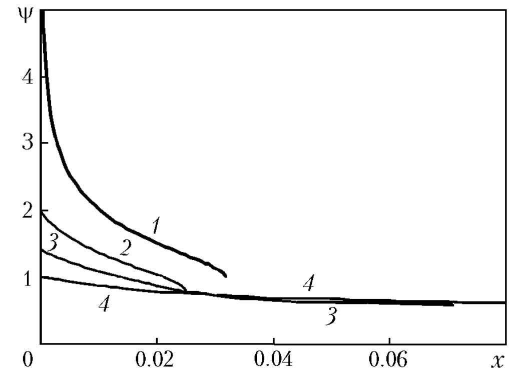


Fig. 5. Distribution of the ratio between the velocity of a traveling wave and the velocity of sound along the variable x at different values of φ_0: 1) $\varphi_0 = 0$ and $\eta(0) = 1$; 2) 16/275 and 0.8; 3) 4/45 and 2/3; 4) 1/8 and 0.5.

determining the ratio between the velocity of the front of a traveling wave and the mass velocity of sound for each of the curves $\eta = \eta(x)$ shown in Fig. 4.

As already noted, the problem being considered is solved at $\varphi_0 = 0$ on condition that $D \geq \rho C_\gamma$ (see curve 1 in Fig. 5). In the case of hyperbolic heat transfer where $0 < \varphi_0 < \varphi_0^{\text{cr}}$, the solution of the type of a traveling wave exists only in the limited range of change in x $0 \leq x \leq x_1$ (curve 2 in Fig. 4). However, unlike the case where $\varphi_0 = 0$, in the regions where the solution exist, the velocities D and ρC_γ become subsonic. At $\varphi_0 > \varphi_0^{\text{cr}}$ (curves 3 and 4 in Figs. 4 and 5) the solution with a discontinuity at the point $x = 0$ exists at all the values of the variable $x \geq 0$. In this case, in a larger part of the region of existence of the solution there takes place a subsonic heat transfer. For example, for curve 4, $D = \rho C_\gamma$ only at $x = 0$. In the region of $x > 0$, $D < \rho C_\gamma$.

Traveling waves with an "internal" discontinuity. As was noted above, in the case where a heat flow is defined by the Fourier law, in the region of existence of a continuous solution of the problem being considered, there takes place a supersonic heating: $D \geq \rho\sqrt{RT}$. In the case where the Fourier law is fulfilled, the authors of [1, 7] have gone to $D < \rho\sqrt{RT}$ by introduction of discontinuities at any points inside the region of existence of the solution. At $\tau \equiv 0$, the discontinuity is isothermic.

Below is presented an example of the distribution of the function $\eta = \eta(x)$ with the above-mentioned "internal" discontinuity in the case of hyperbolic heat transfer at $\varphi_0 \neq 0$.

Let the values of the desired functions be determined at the point $x = x_1$, $0 < x_1 < \infty$:

$$\alpha = \alpha_1\,, \quad \eta = \eta_1\,, \quad f = f_1\,, \quad \beta = \beta_1\,, \quad \omega = \omega_1\,, \quad \tilde{V} = \tilde{V}_1\,. \tag{58}$$

In the case where equalities (58) are fulfilled, the constants in (41) will take the form $C_0 = \eta_1 + \alpha_1$, $C_1 = \alpha_1 - \beta_1$, $C_2 = \frac{1}{\gamma - 1} f_1 + \frac{1}{2}\alpha_1^2 - \beta_1\alpha_1 - \omega_1$. As a result, the desired functions are determined by the formulas

$$\alpha = \eta_1 + \alpha_1 - \eta\,, \quad \beta = \alpha - \alpha_1 + \beta_1 = \beta_1 + \eta_1 - \eta\,,$$
$$f = \beta\eta = \eta\,(\beta_1 + \eta_1 - \eta)\,, \quad \omega = \omega_1 + (\eta_1 - \eta)\left[-\frac{\gamma}{\gamma - 1}\beta_1 + \frac{\gamma + 1}{2\,(\gamma - 1)}\left(\eta - \frac{\gamma - 1}{\gamma + 1}\eta_1\right)\right]. \tag{59}$$

Let us write the relations for the discontinuity front (32) in variables (37)

$$\eta_2 - \eta_1 = \alpha_1 - \alpha_2\,, \quad \alpha_1 - \alpha_2 = \beta_2 - \beta_1\,,$$

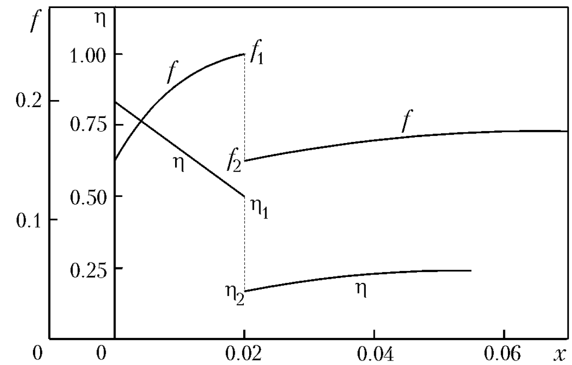


Fig. 6. Profiles of the functions $\eta = \eta(x)$ and $f = f(x)$ in the region of $x \geq 0$. Both functions have two discontinuities at $x = 0$ and $x = x_1 = 0.0178$.

$$\frac{1}{\gamma-1}\eta_2\beta_2 + \frac{1}{2}\alpha_2^2 - \frac{1}{\gamma-1}\eta_1\beta_1 - \frac{1}{2}\alpha_1^2 = \beta_2\alpha_2 - \beta_1\alpha_1 + \omega_2 - \omega_1\,, \quad \tilde{V}_2 - \tilde{V}_1 = \omega_2 - \omega_1\,.$$

On rearrangement, we obtain

$$\alpha_2 = \alpha_1 + \eta_1 - \eta_2\,, \quad \beta_2 = \beta_1 + \eta_1 - \eta_2\,, \quad f_2 = \beta_2\eta_2 = \eta_2(\beta_1 + \eta_1 - \eta_1)\,,$$
$$\omega_2 = \omega_1 + (\eta_1 - \eta_2)\left[\frac{1}{2}\frac{\gamma+1}{\gamma-1}\left(\eta_2 - \frac{\gamma-1}{\gamma+1}\eta_1\right) - \frac{\gamma}{\gamma-1}\beta_1\right], \quad \tilde{V}_2 - \tilde{V}_1 = \omega_2 - \omega_1\,. \tag{60}$$

For the above-presented distributions of the functions $\eta = \eta(x)$ at $a_0 = 2$ and $a_1 = b_0 = b_1 = 1$ (see Fig. 4), the solution of Eq. (53) at the boundary condition $\eta = 1$, $\tilde{V} = 0$ is determined by the formula

$$\tilde{V} = \frac{1}{2\varphi_0}\eta^2(1-\eta)^2\,. \tag{61}$$

The relation $\tilde{V}_2 - \tilde{V}_1 = \omega_2 - \omega_1$ can be written, in view of (61), for the values of the quantities upstream of the discontinuity front ($\eta = \eta_1$, $\varphi_0 = \varphi_{01}$) and downstream of it ($\eta = \eta_2$, $\varphi_0 = \varphi_{02}$) in the following form:

$$\frac{1}{2\varphi_{02}}\eta_2^2(1-\eta_2)^2 - \frac{1}{2\varphi_{01}}\eta_1^2(1-\eta_1)^2 = \omega_2 - \omega_1\,. \tag{62}$$

We now consider the curves constructed in Fig. 4 for the case where $\varphi_0 \neq 0$. Let a portion of any of this curves corresponds to the range of change in the independent variable $0 \leq x \leq x_1$ at a given value of $\varphi_0 = \varphi_{01}$. Since, in the indicated region, the desired functions are determined by formulas (45), it may be assumed that $\beta_1 = 1 - \eta_1$ in (60). In this case, relation (62) will take the form

$$\frac{1}{2\varphi_{02}}\eta_2^2(1-\eta_2)^2 = \frac{1}{2\varphi_{01}}\eta_1^2(1-\eta_1)^2 + (\eta_1 - \eta_2)\left[-\frac{\gamma}{\gamma-1} + \frac{\gamma+1}{2(\gamma-1)}(\eta_1 + \eta_2)\right]. \tag{63}$$

It follows from (63) that

$$\varphi_{02} = \frac{\eta_2^2(1-\eta_2)^2}{\frac{1}{\varphi_{01}}\eta_1^2(1-\eta_1)^2 + 2(\eta_1 - \eta_2)\left[-\frac{\gamma}{\gamma-1} + \frac{\gamma+1}{2(\gamma-1)}(\eta_1 + \eta_2)\right]}\,. \tag{64}$$

We now consider the distribution of the function $\eta = \eta(x)$ for $\varphi_0 = 16/275 \simeq 0.058182$ (curve 2 in Fig. 4). Let an "internal" discontinuity arise at $x = x_1 \simeq 0.0178$, $\eta(x_1) = \eta_1 \simeq 0.55$, $\eta(x_1) = \eta_2 \simeq 0.2$. On the assumtion that $\varphi_{01} = \varphi_0$ and $\gamma = 5/3$, by formula (64) we determine the corresponding value of the parameter φ_{02}: $\varphi_{02} \simeq 0.072554$. It is assumed that $\varphi_{02} > \varphi_0^{\text{cr}} = 0.0625$. At $x > x_1$, the distribution $\eta = \eta(x)$ exists at all the values of $x > 0$, and $\eta \to \frac{\gamma - 1}{\gamma + 1} = 0.25$ at $x \to \infty$. Figure 6 presents the distributions of the function $\eta = \eta(x)$, defining the dimensionless specific volume, and the dimensionless temperature function $f = f(x)$ in the region of $x \geq 0$; each of these functions has two discontinuities at $x = 0$ and $x = x_1 = 0.0178$.

CONCLUSIONS

The hyperbolic heat transfer depends substantially on the change in the relaxation time τ: an increase in it leads to an increase in the absolute temperature and to a decrease in the depth of heating.

The traveling waves defining the heat transfer by the Fourier law ($\tau \equiv 0$) exist only at values of the independent variables m and t changing within limited ranges, in which the velocity of the traveling wave is larger than the velocity of sound. In the case where $\tau \neq 0$, a solution of the type of a traveling wave can exist at all the values of $m \geq 0$ and $t \geq 0$, and the "supersonic" flow is changed to a "subsonic" one.

A hyperbolic heat transfer is characterized by the change from the "initial background" in the form of a large discontinuity of the desired functions, including the temperature and heat-flow functions. In this case, traveling waves with the second ("internal") discontinuity of quantities also exist.

It is shown that the solutions being considered differ substantially from the classical ones that are considered on the assumption that the heat flow is proportional to the temperature gradient (Fourier law).

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NOTATION

C, C_0, C_1, C_2, integration constants; C_γ, isothermal velocity of sound; D and $\overline{D}$, mass and constant mass velocities of the front of a large discontinuity of the desired quantities respectively; D_T, mass velocity of propagation of thermal disturbances including the relaxation of the heat flow; F, $\underline{F}$, desired functions; $f = f(s)$, dimensionless function of the temperature variables s; $K = K_0 T^{a_0}$, mass coefficient of heat conductivity represented as the temperature power function; K_0, constant; m, Lagrange mass coordinate; p, pressure function; R, universal gas constant; s, dimensionless "self-similar" variable; s_0, dimensionless "self-similar" variable defining the boundary condition; t, time; T, $\tilde{T}$, temperature function; v, velocity function; V, function determined by relation (12); $\hat{V}$, function determined in the group of formulas (37); $\tilde{V}$, particular case of the function V; W, $\tilde{W}$, functions of heat flow; W_0, initial value of the function of heat flow; x, dimensionless variable for the case of traveling waves; $\alpha = \alpha(s)$, dimensionless function of the velocity variable s; $\beta = \beta(s)$, dimensionless function of the pressure variable s; $\gamma > 1$, constant relation between the specific heat capacities; ε, specific internal energy; $\eta = \eta(s)$, specific volume representing the dimensionless function of the density variable s; $\tilde{\eta}$ and $\hat{\eta}$, special cases of the dimensionless function η; ρ, density function; ρ_0, initial value of the density function; $\tau = \tau_0 T^{a_1}$, relaxation time of a heat flow represented as the temperature power function; $\hat{\tau}_0$ and φ_0, dimensionless combinations of the constants τ_0, R, and ρ_0; φ_0^{cr}, critical value of φ_0; ψ, function determined by relation (57); $\omega = \omega(s)$, dimensionless function of the heat-flow variable. Subscripts: *, selected value of a function at $m = 0$; 1, value upstream of the discontinuity front; 2, values downstream of the discontinuity front; a_0, coefficient defining the temperature dependence of the heat-conductivity coefficient; a_1, coefficient defining the temperature dependence of the relaxation time; b_0, coefficient defining the dependence of the heat conductivity on the density; b_1, coefficient defining the dependence of the relaxation time on the density; g, coefficient defining the time dependence of the heat flow; n, coefficient defining the time dependence of the dimensionless "self-similar" variable; n_0, coefficient defining the time dependence of the dimensionless temperature function; v, vacuum; cr, critical.

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